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The acceptability of papers offered must largely depend upon the judgments of our editorial referees. Final decisions will be made as promptly as possible and reported to those concerned.

This Journal is dedicated to the following aims:

1. THROUGH PUBLISHED STANDARD PAPERS ON THE CULTURE ASPECTS, HUMANISM AND HISTORY OF MATHEMATICS TO DEEPEN AND TO WIDEN PUBLIC INTEREST IN ITS VALUES.
2. TO SUPPLY AN ADDITIONAL MEDIUM FOR THE PUBLICATION OF EXPOSITORY MATHEMATICAL ARTICLES.
3. TO PROMOTE MORE SCIENTIFIC METHODS OF TEACHING MATHEMATICS.
4. TO PUBLISH AND TO DISTRIBUTE TO THE GROUPS MOST INTERESTED HIGH-CLASS PAPERS OF RESEARCH QUALITY REPRESENTING ALL MATHEMATICAL FIELDS.

A Startling Paradox

I am wondering if the mathematicians should not recommend that the mathematics of the elementary and secondary schools be moved, bodily, a year or two, later in the curriculum. If complaint is then made (as it will be in some cases) that the work is needed earlier, for prerequisites and other reasons, we can reply that it is better to get it later than not at all.

Here is the situation as I see it: Mathematics is under fire because it is hard and because, being hard, pupils do not get as much out of it as they should for the time and energy used. One chief reason why it is hard is that we teach it to our students before they are mentally ready to cope with it adequately. One reason why we teach it to them too early is that it has always been considered so important that it has been felt that they *must* get it, and get it as early as it is possible for them to receive it. Hence we are arriving at the argument (which is more logical than it sounds when put in this way) that we must throw out mathematics because it is so very important.

Chas. H. Sisam, Colorado College.

Concerning Preparedness for College Algebra at the University of South Carolina

By J. Bruce Coleman
University of South Carolina

I.

Percentage of students making different grades in college algebra depending upon whether or not the time intervening between the study of high school algebra and of college algebra was less than one year. (No account is taken of whether the last course in high school algebra was review or advanced.)

Group	*Interval*	*No. in Group*	*% of Group Making* A	B	C	D	E	W
1	Less than one year	205	14	18	20	18	20	10
2	One year or more	115	7	8	16	17	38	24

For group 1 the Pearson product-moment coefficient of correlation between grades in college algebra and grades in other college subjects was found to be .14 ± .04. For group 2, the same coefficient was found to be .34 ± .05.

II.

Relative percentages of students in the freshman class, and their standing in college algebra, with respect to the size of the high schools in which they were prepared. Group 1 comprises high schools in which the enrollments were from 1481 to 376; in group 2 they were from 375 to 179; in group 3 they were 178 or less.

Group	*% of H. S. Population of State*	*% of Freshman Class from Group*	*% of Group Making* A	B	C	D	E	W
1	25	53	13	17	21	17	22	10
2	25	22	10	16	19	17	23	15
3	50	25	10	7	15	18	28	22

Note:—Grades of A, B and C are considered satisfactory. A grade of D implies semester credit without honor points, so might be considered a partial success. E is a complete failure, and W indicates withdrawal from the course before its completion. A few of the

grades of E should perhaps be classed as W, since they were incurred as the penalty for unexcused absences. Should these be transferred, the effect would probably be negligible as regards any conclusions.

Data for 1932-33, 1st semester.

Fermat and Euler

By James McGiffert

Students of the Theory of Numbers know that during almost countless ages men have sought for a definite formula which would determine the prime or composite nature of a given number, but all to no avail. Tables giving lists of primes up to many millions have been constructed.

The mathematician, Fermat, thought he could assert that a number of the form $2^{2^n}+1$ was prime, although he confessed that he had not been able to prove it. Soon Euler came forward with the statement that when $n=5$, the number was divisible by 641. That is, $2^{32}+1$ is divisible by 641.

Anyone can prove this to be true by raising 2 to the 32nd power, adding 1, and dividing the resulting number by 641. The quotient will be found to be integral. For some time I have wondered if I might not find a simple, short method of showing the truth of this solution which would not involve laborious long division. Just a day or two ago I devised this solution.

$$641=5\ .\ 128+1=5.2^7+1.$$

The highest exponent of 2, less than 32, which is a multiple of 7, is 28. Hence I reduced 2^{32} to $2^4.2^{28}=16.2^{28}$.

Hence we have to divide $16.2^{28}+1$ by 5.2^7+1. Dividing $16.2^{28}+1$ by 5.2^7+1, we get an integral quotient of 3.2^{21} with a remainder of $2^{28}-3.2^{21}+1$

But $2^{28}=2^7.2^{21}=128.2^{21}$. Hence our remainder is

$$128.2^{21}-3.2^{21}+1=125.2^{21}+1.$$

We see at once that $125.2^{21}=(5.2^7)^3$. We see that our division therefore reduces to that of dividing x^3+1, by $x+1$, and we know that the quotient is x^2-x+1, and we conclude that

$$125.2^{21}+1 \text{ divided by } 5.2^7+1 \text{ yields}$$

$$25.2^{14}-5.2^{7}+1, \text{ and the division is exact.}$$

Collecting our results, we find that the quotient obtained after dividing

$$2^{32}+1 \text{ by } 641 \text{ is}$$

$$3.2^{21}+25.2^{14}-5.2^{7}+1,$$

which is evidently an integer.

The Trigonometry Based on a Central Conic

By H. L. Smith

In the issue of the *Mathematics News Letter* for April, 1931, we introduced certain functions defined with reference to the central conic

(1) $$x^2+2k\,x\,y+l\,y^2=1$$

and which we called *conic* functions. Concerning two of them, cosc t, sinc t we proved the following formulas:

(2) $$\operatorname{cosc}^2 t+2\,k \operatorname{cosc} t \operatorname{sinc} t+l \operatorname{sinc}^2 t=1$$

(3) $$\operatorname{cosc} 0=1,\ \operatorname{sinc} 0=0$$

(4) $$\begin{cases} D \operatorname{cosc} t=-k \operatorname{cosc} t-l \operatorname{sinc} t \\ D \operatorname{sinc} t=\operatorname{cosc} t+k \operatorname{sinc} t \end{cases}$$

(5) $$\begin{cases} \operatorname{cosc}(a+b)=\operatorname{cosc} a \operatorname{cosc} b-l \operatorname{sinc} a \operatorname{sinc} b \\ \operatorname{sinc}(a+b)=\operatorname{sinc} a \operatorname{cosc} b+\operatorname{cosc} a \operatorname{sinc} b+2k \operatorname{sinc} a \operatorname{sinc} b \end{cases}$$

(6) $$\begin{cases} \operatorname{cosc}(a-b)=\operatorname{cosc} a \operatorname{cosc} b+2k \operatorname{coasc} b+l \operatorname{sinc} a \operatorname{sinc} b \\ \operatorname{sinc}(a-b)=\operatorname{sinc} a \operatorname{cosc} b-\operatorname{cosc} a \operatorname{sinc} b \end{cases}$$

(7) $$\begin{cases} \operatorname{cosc}(-b)=\operatorname{cosc} b+2k \operatorname{sinc} b \\ \operatorname{sinc}(-b)=-\operatorname{sinc} b \end{cases}$$

In the present note we develop further formulas.

1. **The general case.** If we regard (5),(6) as simultaneous equations and solve for cosc a cosc b, etc., we get the following:

(8) $$2(l-k^2)\operatorname{sinc} a \operatorname{cosc} b=l \operatorname{sinc}(a+b)+(l-2k^2) \operatorname{sinc}(a-b)$$

$$+k \operatorname{cosc}(a+b) - k \operatorname{cosc}(a-b)$$

(9) $2(l-k^2) \operatorname{sinc} a \operatorname{sinc} b = -k \operatorname{sinc}(a+b) + k \operatorname{sinc}(a-b) - \operatorname{cosc}(a+b) + \operatorname{cosc}(a-b)$

(10) $2(l-k^2) \operatorname{cosc} a \operatorname{sinc} b = l \operatorname{sinc}(a+b) - l \operatorname{sinc}(a-b) + k \operatorname{cosc}(a+b) - k \operatorname{cosc}(a-b)$

(11) $2(l-k^2) \operatorname{cosc} a \operatorname{cosc} b = -lk \operatorname{sinc}(a+b) + lk \operatorname{sinc}(a-b) + (l-2k^2) \operatorname{cosc}(a+b) + l \operatorname{cosc}(a-b)$

If in formulas (8),(9),(11) we put $b=a$ we get

(12) $2(l-k^2) \operatorname{cosc}^2 a = (l-2k^2) \operatorname{cosc} 2a - lk \operatorname{sinc} 2a + l$

(13) $2(l-k^2) \operatorname{sinc}^2 a = 1 - \operatorname{cosc} 2a - k \operatorname{sinc} 2a$

(14) $2(l-k^2) \operatorname{cosc} a \operatorname{sinc} a = k \operatorname{cosc} 2a + l \operatorname{sinc} 2a - k$

From (4) we get the following formulas:

(15) $D^{2m+1} \operatorname{sinc} t = (k^2-l)^m(\operatorname{cosc} t + k \operatorname{sinc} t)$

(16) $D^{2m} \operatorname{sinc} t = (k^2-l)^m \operatorname{sinc} t$

(17) $D^{2m+1} \operatorname{cosc} t = (k^2-l)^m(-k \operatorname{cosc} t - l \operatorname{sinc} t)$

(18) $D^{2m} \operatorname{cosc} t = (k^2-l)^m \operatorname{cosc} t$

From (15)-(18) we get the following Maclauren series:

(19) $\operatorname{sinc} t = t + (k^2-l)t^3/3! + (k^2-l)^2t^5/5! + \dots$

(20) $\operatorname{cosc} t = 1 + (k^2-l)t^2/2! + (k^2-l)^2t^4/4! + \dots - k[t + (k^2-l)t^3/3! + (k^2-l)^2t^5/5! + \dots].$

From (19),(20) we have

(21) $\operatorname{sinc}(t, 0, l) = t - lt^3/3! + l^2t^5/5! - l^3t^7/7! + \dots$

(22) $\operatorname{cosc}(t, 0, l) = 1 - lt^2/2! + l^2t^4/4! - \dots$

From (21),(22) we get

(23) $\operatorname{sinc}(t, k, l) = \operatorname{sinc}(t, 0, l-k^2)$

(24) $\operatorname{cosc}(t, k, l) = \operatorname{cosc}(t, 0, l-k^2) - k \operatorname{sinc}(t, 0, l-k^2)$

Also by (21),(22),

(25) $\operatorname{sinc}(pt, 0, l) = p \operatorname{sinc}(t, 0, lp^2)$

(26) $\operatorname{cosc}(pt, 0, l) = \operatorname{cosc}(t, 0, lp^2)$

It now follows from (23)-(26) that

(27) $\operatorname{sinc}(t, k, l) = [1/\sqrt{|1-k^2|}] \operatorname{sinc}(t\sqrt{|1-k^2|}, 0, \operatorname{sgn}(1-k^2))$

(28) $\operatorname{cosc}(t, k, l) = \operatorname{cosc}(t\sqrt{|1-k^2|}, 0, \operatorname{sgn}(1-k^2))$

$$-[k/\sqrt{|1-k^2|}] \operatorname{sinc}(t\sqrt{|1-k^2|}, 0, \operatorname{sgn}(1-k^2))$$

We also note the formula

(29) $(\operatorname{cosc} t + j \operatorname{sinc} t)(\operatorname{cosc} u + j \operatorname{sinc} u)$

$$= \operatorname{cosc}(t+u) + j \operatorname{sinc}(t+u)$$

where

(30) $j = k + \sqrt{(k^2-1)}$.

From it the reader will easily obtain a generalization of de Moivre's theorem.

2. **The elliptic case.** Let us now consider the case in which

(31) $1-k^2>0$.

In this case we have by (27),(28)

(32) $\operatorname{sinc}(t, k, l) = [1/\sqrt{(1-k^2)}] \sin t\sqrt{(1-k^2)}$

(33) $\operatorname{cosc}(t, k, l) = \operatorname{cosc} t\sqrt{(1-k)^2} - [k/\sqrt{(1-k^2)}] \operatorname{sinc} t\sqrt{(1-k^2)}$.

But (33) may be put into the form

(34) $\operatorname{cosc}(t, k, l) = \sqrt{[l/(1-k^2)]} \cos (t\sqrt{(1-k^2)} - \arcsin [-k/\sqrt{l}])$

By aid of (32),(34) we see at once that sinc t, cosc t are both periodic of period $2\pi/\sqrt{(1-k^2)}$, that each function has one maximum and one minimum in a period.

A Letter to the Editor

Urbana, Ill., Nov. 17, 1934

My dear Professor Sanders:

I was much pleased to see that the *National Mathematics Magazine* aims to publish papers on the history of mathematics since it seems to me that this subject represents at the present time one of the weakest parts of American mathematics. During the last decade very rapid advances have been made in this subject, especially as regards very ancient mathematics. Hence the text books on this subject usually give inadequate accounts relating thereto even if they were approximately up to date at the time of publication. For instance, recent discoveries relating to the finding of at least one root by the ancient Babylonians of certain numerical quadratic and cubic equations throws new light on the history of algebra and on the contributions made by the Greeks and the Arabians towards the solution of algebraic equations.

The methods used by the ancient Babylonians to solve their quadratic equations seem to have been practically the same as those employed at the present time but as regards cubic equations they proceeded in a very different manner than we do to-day. They constructed tables of numbers of the form n^2+n^3 for the different values of n and then reduced their cubic equations to the form $x^3+x^2=a$. The given tables then enabled them to find a real definite root, at least approximately, when such a root exists. It is to be emphasized that the complete solution of general quadratic and of general cubic equations could not be attained until our ordinary complex numbers began to be understood at about the beginning of the nineteenth century, although complete formal solutions were used earlier in Europe. It is therefore far from the truth to say that "the general quadratic as we know it today was thus fully mastered by Greek mathematicians"—D. E. Smith, *History of Mathematics,* Volume 1 (1923), page 126.

The student of the history of mathematics naturally desires to correct his own books on this subject as errors are reported and many of them doubtless regret that what is now the most extensive work on this general subject in the English language was not reviewed in the *Bulletin of the American Mathematical Society,* where one might have expected to find references to desirable modifications. Historical knowledge cannot be expected to grow vigorously unless statements which appear to be erroneous are considered on their own merits irrespective of where they may have first appeared. Such considerations may sometimes exhibit the fact that they are not as unsound as they at first appeared to be. With respect to the extensive historical writings of

the late Florian Cajori it seems to me that R. C. Archibald brought out an important feature when he said: "Many of Professor Cajori's publications, especially in the pre-Californian days, show evidence both of haste in composition and of lack of checking in proof with the sources of information." *Isis,* volume 17 (1932), page 388.

This really means that the reader should carefully check the statements which he finds in Cajori's writings before he accepts them as commonly accepted historical facts even at the time when they were published. Probably the most reliable place to do this at the present time is the *Geschichte der Elementar-Mathematik* by J. Tropfke, in seven volumes, of which the first two have appeared in the third edition 1930 and 1933, respectively. The main object of this letter is to emphasize the fact that historical mathematics writings should be prepared with the most utmost care in order to be really useful. Too many of them are based on statements which are misleading. One paper of this type, which appeared in the *American Mathematical Monthly,* was recently reviewed by the following sentence: "A series of assertions are not correct." *Zentralblatt fur Mathematik,* vol. 9 (1934), page 97.

G. A. MILLER, University of Illinois

Mathematical Knowledge

"Still, although mathematical knowledge does not lead to absolutely certain results, it yet invests known results with incomparably greater trustworthiness than does the knowledge of the other sciences. But after all, it remains a useless accumulation of capital so long as it is not turned to practical account in other sciences, such as metaphysics, physics, chemistry, biology, political economy, etc. Hence also arises an obligation on the part of the other sciences, so to shape their problems and investigations that they can be made susceptible of mathematical treatment. Then will mathematics gladly perform her duty. The moment a science has advanced far enough to permit of the mathematical formulation of its problems, mathematics will not be slow to treat and to solve these problems. Mathematical knowledge, aristocratic as it may appear, by the greater certainty of its results, will, so far as the advancement of human kind is concerned, never be more than a useless mass of self-evident truths, unless it constantly places itself in the service of the other sciences." —"On the Nature of Mathematical Knowledge," by Hermann Schubert.

The Teachers Department

Edited by

JOSEPH SEIDLIN AND W. PAUL WEBBER

*A PROPOSAL FOR THE IMPROVEMENT OF THE TEACHING OF MATHEMATICS

No doubt there is great need for improvement in the teachers and in the teaching of mathematics. The universities, our own Ohio colleges, for example, have found their incoming students woefully deficient in their ability to cope with elementary arithmetic and algebraic technique, as well as with simple common-sense problems. Naturally, the colleges blame the teachers in the secondary schools. The algebra teachers, in turn, find that the students coming to them are badly prepared and, in their efforts to cover the course of study of the high school, have no time to correct and to supplement these deficiencies, and so down the line from one grade to the lower one, until it becomes manifest that if there is to be any improvement in the teaching of mathematics, it must begin in the very lowest grade and continue throughout the entire college span.

Mathematics has become so unpopular with educators that they are advocating its removal as a required subject for graduation from the high schools in our country. Thus for the past year the Cincinnati schools have not required Plane Geometry, and this year, according to the plan presented for approval to the state supervisor, Algebra is also to be taken off the required list. A student may now graduate from an accredited high school in Cincinnati without having had any mathematics whatever. While this trend may be due in part to the inherent difficulties of mathematics for many students, I believe that most of the trouble may be attributed to the poor teaching.

Much of the instruction given to our students throughout the entire educational system has been poor both in emphasis and in methods of presentation. The responsibility for this rests on the shoulders of the mathematicians themselves. For, as I shall point out, it is our duty in the colleges to provide adequate means for all students who are contemplating teaching as a profession to acquire the proper training. And, unless the mathematicians themselves take steps to remedy matters, we have no right to voice complaints.

It is evident that the greater emphasis which is being placed at present on methods of teaching at the expense of the content of the

*Chairman's Address, at the annual meeting of the Ohio Section of the Mathematical Association of America, held in Columbus, Ohio, April 5, 1934.

subject matter has to a large extent contributed to the lamentable status of mathematics. On the other hand, it is also true that even an excellent knowledge of the subject matter is not sufficient to insure good teaching at the outset of the teacher's career. To be sure, the prospective teacher profits from taking more and more courses in mathematics, but at no time does he have the opportunity to pause and to consider the development of the subject and the unification of the various courses. And when is he encouraged to take stock of himself and to think about the presentation of the subjects which he will at some time be called upon to teach—not a formal presentation, but one which is natural and stimulating?

I am not concerned here with the pros and cons of a unified freshman course, but I am interested in the actual presentation of the fundamental ideas of the mathematics usually taught in the schools and colleges. We all know persons with master's and with doctor's degrees in mathematics, who have had many troublesome experiences on their first teaching jobs, at the expense of their students. And, if people who are so well informed have these difficulties, those with a poorer mathematics background will surely find it hard to give their students the proper inspiration and love for the subject which ought to be a challenge to the intelligence of every person.

Instead of limiting ourselves to complaining about the courses offered by schools of education, which are usually not in a position to employ specially trained mathematicians to give the proper orientation to the future teachers, let us do something constructive. We need only take advantage of the facilities we already have, and need go to no further expense to help the future teacher avoid these pitfalls.

There are of course many people who believe that it is impossible to train teachers. Their contention is that good teachers are born, not made. If we accept this premise, then there is little hope for any improvement, for there are very few among us who are *born* teachers. There are still others who claim that there is no need for teacher training, because the students absorb methods of presentation from their own teachers. Undoubtedly one profits from a good presentation of the subject matter, but the student is so concerned with the content of the course that he cannot be expected to focus his attention on an analysis of the methods the instructor uses in his teaching. I personally entertain the hope that the teaching of mathematics can be improved and that we have it in our power to do it effectively.

There are two classes of students for whom we must make provision, viz.,

1. Those who expect to teach in the elementary schools, but whose major interest is not mathematics.

2. Those who plan to major in mathematics and who will eventually teach in the secondary schools, or in colleges.

For the first group of students I propose a required course in mathematics which will have for its aims:

(a) An understanding of arithmetic apart from the technique. For example, the meanings of the operations, and the logical development of the rules.

(b) An account of the difficulties which the human race has had in grappling with the concepts of the various types of numbers; of more elementary portions of algebra, especially of those portions which will throw light on the true meaning of arithmetic.

(c) Actual presentation by the future teachers of some of the fundamental portions of arithmetic as they may be taught to a class in an elementary school. This is to be an application of (a) and (b) and to come simultaneously with them. The presentations will necessarily be informal and natural, rather than formal and elegant, so that the imaginary pupil will see the subject developing before him, and not in its mysterious finished form. Mathematics should and must lose its reputation of being a dead and dreaded science and should emerge as a living, interesting and stimulating activity.

1. Scales of notation other than ten.
2. The advantage of the present positional system of representing numbers over that of other systems such as the ancient Greek and Roman systems.
3. The explanation of indention in the multiplication of whole numbers.
4. Tests for divisibility by 9, 11, etc.
5. Casting out nines and applications.
6. Meaning of fractions and logical basis behind the operations.

Many educators believe it is enough to give students the technique in the hope that some day those who are really interested will be curious enough to look for explanations. But this day never seems to arrive in the lives of most students. In fact, some educators go as far as to say it is not possible to teach anything of the meaning of arithmetic to young students, so that teachers are discouraged from making any attempts in this direction. As a student advances in his school work he becomes deluged with more and more technique; he becomes so bewildered, that he feels it is hopless to try to understand. From the elementary grades

on he is taught that every problem he meets, which in most cases could be solved by the simplest application of common sense, is to be solved by matching the problem with certain definite operations.

I proposed the following problem to my own students at the University: One clerk can write 100 letters in two hours, and another can do the same work in 4 hours. how long will it take them to write the 100 letters if they work together? The immediate response was 3 hours, because to these students it brought to mind the kind of problem that required averaging. Others, who had already encountered work problems in their high school algebra, immediately took out pencil and paper and started in with x's. Some put the x's in the numerator and others put the x's in the denominator. Even after discussion in class, which brought out the fact that the problem involved only common sense, some students still persisted in applying the heavy artillery of algebra to this problem and not always correctly. In fact, the students had a feeling that they were not doing mathematics because there was no equation involving x. And one semester was not quite long enough to cure all these students of their cubby-hole idea of mathematics, and to restore their confidence in their own good common sense. As a matter of fact, for most students mathematics and common sense are not to be mixed.

Is it any wonder then that educators are convinced that there is no carry-over from mathematics? For, if mathematics means merely a matching of operations with problems, what is there to carry over, since the arithmetical operations and the x's are not present in other fields? Mathematics from the very outset should be the study par excellence, in which the student will have an opportunity to develop his common sense and judgment, and to use a minimum number of tools in arriving at his result, so that if a problem can be solved by arithmetic instead of algebra, the arithmetical solution is preferable. For, as we all know, symbolism is merely an aid in thinking and should be reserved for those problems which involve so many steps that it becomes too difficult to carry them out without the aid of the symbols. We must guard against becoming the slaves of the symbols; rather we must remain masters over them. In other words, we must insist on the use of common sense.

At the University of Cincinnati I have, for the past ten years, given precisely such a course as was just outlined. This is required of *all* students who plan to teach in the elementary grades, and is given by the mathematics department in the Liberal Arts college. Although the course is not intended for those who plan to major in mathematics, each year a number of students who register for this course, develop an

interest in this subject, and take more advanced work. The time is divided about equally between content and method.

Every college has many students who will eventually teach in elementary schools, and who have no special interest in college mathematics. Such students should not be forced to take the conventional courses in Algebra, Trigonometry and Analytics, but should rather be given the oportunity to obtain a thorough grounding in the principles of arithmetic and algebra. This course should be offered in the mathematics department by one of the regular members of the staff, whose particular interest is along these lines.

It will not be easy at first to convince students that this type of course will eventually be of more value to them than the conventional mathematics courses given in the Liberal Arts College. Students have to become accustomed to the idea that learning to understand the principles underlying arithmetic, is as important a study of mathematics as the actual solving of problems. The student gets this idea only after he has himself tried to give an exposition of a simple arithmetical fact. He then realizes how much thought and preparation are required to make such an explanation clear both to himself and to the listener.

The student will eventually begin to enjoy this type of work and make contributions of his own. Once, in my own class, when discussing division of whole numbers as the process of repeated subtraction, I asked a student how she would extend this idea of division to finding the number of portions which one could obtain from four small pies if each portion is to be $\frac{3}{4}$ of a pie. She saw at once that the method of repeated subtraction was still applicable in terms of smaller units, and gave, in addition, a graphical method which was clear and vivid. Certainly such a method would have much more meaning than the rule of inverting the divisor, which the student learns only too soon and never really understands.

Of course the teacher himself will have to do considerable planning and it will not be as easy as teaching a course in trigonometry. As for a basic text for such a course, one could mention several. This may be supplemented with ideas and devices which the teacher himself supplies.

We come next to the second group of students, namely, those who have majored in mathematics, and who will eventually teach in the secondary schools and colleges. The students of this group are with us for several years and therefore we are in a position to orient them properly. After such a student has taken the usual courses in the department, it would be an excellent time for him to review, from the point of view of motivation, and natural methods of presentation, the nature of these courses. I shall give a few illustrations.

Some of the topics of algebra with which every student should be familiar and which should be presented in such courses are:

1. Motivation of problems in parentheses.
2. Introduction of simple problems in permutations and combinations as an aid in the development of reasoning and judgment.
3. Problems leading to simple Diophantine equations and their solution.
4. Motivation of negative and fractional exponents, with a natural exposition of this subject, differing from the formal treatment found in algebra texts.
5. The logic involved in solving quadratic and other equations.
6. A simple method of solving "word" problems, both algebraically and graphically.

Let us consider for example the method of teaching word problems in algebra. Every teacher knows what difficulties students have with such problems. If we examine some of the high school texts and even some of the more recent college algebra texts, we find distinct methods given for the solutions of type problems, such as rate problems, work problems, clock problems, mixture problems, etc. Usually the student sees no single method for dealing with all such types and his only procedure is to classify every problem he meets and use the special method he has memorized for that type.

The solutions of such problems may be made very easy by a simple psychological device. We suppose the problem solved and with the aid of the assumed answer proceed with the actual arithmetical verification. The equation is obtained at once by substituting x for the answer used. One might argue that checking the answer is just as difficult as setting up the equation, but psychologically, it is easier for a student to work with a concrete number than with the x. Not only does this method teach the student to solve such problems more effectively, but it gives him an opportunity to develop his judgement in giving a good estimate of the answer.

It is with geometry that the new teacher has the greatest trouble. Here the subject matter as presented in text books is given in the most unnatural form and the student is so overawed by the smooth and elegant exposition that he gets no clue to its "mysteries," and must therefore rely almost entirely on his memory. I know of no subject in the high school curriculum which is as valuable as geometry for the development of the mental processes. Yet this is one of the most poorly taught of all subjects. I can give at least two reasons for this.

1. Very few teachers have had any training in geometry beyond their own high school course.
2. They have had no opportunity to think about the presentation of the subject.

It is with geometry that the most valuable work can be done for the mathematics majors, not only in giving them further content matter and understanding of synthetic geometry, but also in discussing various means and devices to be used in the teaching of the subject. Thus the student should be made to see that the proposition as stated in the text book is merely the finished form of a chain of reasoning which, when recorded, would bear little resemblance to the proof outlined in his text.

The teaching of trigonometry should at the outset be made less formal. The student is so mystified by the strange sounding words he meets at the very beginning of the course, that he is not able to focus his attention on the fact that he is studying a quantitative aspect of the geometry with which he is already acquainted. Here I would suggest that the names of the trigonometric functions be withheld from the student until he himself sees the need for their introduction.

In analytical geometry we must teach students to view problems both algebraically and geometrically, so that practically each algebraic concept and operation will be seen to have its geometric counterpart, and vice versa. A student does not usually learn this from his first course in analytical geometry, where he is engaged in technical reductions of equations of the second degree. Unless these "counterpart" ideas are emphasized sufficiently, he will find it very difficult to teach this subject properly.

These are just a few of the topics to be considered in the kind of course I am recommending, and if space permitted, I could include many more actual examples; but I hope that what I have said will make my plan clear.

To summarize: We may go a long way in the improvement of the teaching of mathematics in the elementary and secondary schools, if we insist that all students who plan to teach shall take one of the two courses outlined. In the first course, which is primarily for those who plan to teach in the elementary schools, a student is to have the opportunity to learn some of the fundamentals of arithmetic and algebra, which he may be expected to teach in the first eight grades. While the emphasis in this course is on presentation, of necessity, the meanings of the concepts and operations will be considered, along with the historical background. This course should be given in the freshman year, for two semesters, with three meetings a week.

The second course should be required of every mathematics major and should also be given by the mathematics department of the Liberal Arts college. In this course there should be a review of the high school and college mathematics through the calculus, which eventually, he may be called upon to teach. The review should not in any sense be a repetition of any of these courses, but rather a presentation of actual topics by students of the class. Such presentations will again bring up the question of precise meanings of the concepts. Here the student will have a chance to obtain a grasp of the fundamentals of algebra, geometry and the calculus, which will be invaluable to him in his later work. This course should be given throughout the senior year, meeting once or twice a week.

In conclusion I would say that if we make mathematics understandable and natural it will of itself become more attractive and useful to students. It would not take long to restore the subject of mathematics to its important position in the public school curriculum.

I. A. BARNETT, *University of Cincinnati*

The Compelling Power of Mathematics

"Now mathematicians are everywhere aware that their science has a most pronounced habit of producing its most important results without seeking permission anywhere else, without ever asking "by your leave." Indeed so striking is this characteristic that at least once in a generation something is produced by their science which causes mathematicians themselves to view with alarm what is happening. It was true in the days of Pythagoras: and whether we consider the appearance of minus, zero and the imaginaries, the appearance of the non-Euclidean geometries of the last century, or that of the non-Riemannian geometries of this century, it is always the algebraic, the geometric, the analytic, in its own right that is found forcing its way forward, and coming to prevail despite all clamor." —Linguistic Analysis of Mathematics, by Arthur F. Bentley.

William Benjamin Smith

"Nevertheless one cannot contemplate the amount and the quality of William Benjamin Smith's activity without perceiving that the mind o fthis man—mathematician, physicist, poet, philosopher, teacher and critic—was truly Leibnitzian in scope and versatility, in the exactness, the depth, and the immensity of his scholarship, in the catholicity and elevation of his spirit, and especially in the constructive achievements of his critical work. Let us learn at least a little about this extraordinary man, who, though long well known in the chief critical centers of the world, was known and appreciated by only a few of his fellow Countrymen." —By Cassius Jackson Keyser, in Scripta Mathematica.

Notes and News Department

Edited by
I. Maizlish

The Louisiana-Mississippi Section of the Mathematical Association of America and the Louisiana-Mississippi Branch of the National Council of Teachers of Mathematics

The eleventh annual meeting of the Louisiana-Mississippi Section of the Mathematical Association of America and the seventh annual meeting of the Louisiana-Mississippi Branch of the National Council of Teachers of Mathematics were held jointly in Jackson, Mississippi, March 23, 24 with Millsaps College, Belhaven College and Central High School. Dr. P. K. Smith presided over the meetings of the Association and Mr. Henry F. Schroeder presided over the meetings of the Council.

Dr. D. M. Key, president of Millsaps College gave the address of welcome at the opening meeting and Dr. G. T. Gillespie, president of Belhaven College welcomed the group at the dinner Friday evening at Belhaven College. Responses were given by Dr. W. V. Parker, and Mr. Henry F. Schroeder.

The attendance was eighty-five including twenty-seven members of the Association and seven members of the Council.

Prof. Arnold Dresden spoke at the dinner on the subject "The Mathematical Association of America and American Mathematics." He says, "A discussion of the critical situation which faces American mathematics formed the topic for this address. The responsibility of the teachers of mathematics, the measures taken by the Mathematical Association of America to understand and deal with the problems raised by present conditions were brought forward. The audience was requested to join in the work of the Association."

Besides bringing us very helpful addresses Prof. Dresden frequently entered the discussions which were unusually fine at these meetings.

The program was published in the February, 1934, "News Letter". Papers added at the time of the meeting are as follows:

1. "Certain problems on characteristic exponents in the problems of mechanics" by Professor H. E. Buchanan, Tulane University. By title.

2. "Problems of the calculus of variations with higher derivatives in the integrand" by Professor W. L. Duren, Tulane University. By title.

3. "A generalized helium atom problem" by Doctor J. F. Thomson, Tulane University. By title.

4. "On a special type of problem of pursuit" by Mr. G. F. Cramer, Tulane University.

5. "On a unifying theorem in modern geometry" by Professor C. D. Smith, Mississippi State College.

A report of the meeting including abstracts of the papers read before the Association has appeared in the *Mathematical Monthly.*

The meetings next year are to be held with Louisiana College at Pineville, Louisiana. The following year, 1936, they will be at Hattiesburg.

Officers for the Section of the Association were elected; chairman, T. A. Bickerstaff, University of Mississippi; vice-chairman for Mississippi, Dewey Dearman, State Teachers College at Hattiesburg; vice-chairman for Louisiana, V. B. Temple, Louisiana College; secretary-treasurer, Mrs. A. P. Daspit, Louisiana State University.

Officers for the Council are as follows: Chairman, James Beckett, Central High School, Jackson; vice-chairman, Miss A. Foote, Hattiesburg; secretary, Loreen Dyson, Bolton High School, Alexandria, Louisiana.

DOROTHY McCOY,
Secretary of the Louisiana-Mississippi Section of the Association.

Tentative Program for the Pittsburgh Meeting of the National Council of Teachers of Mathematics
Pittsburgh, December 28-29, 1934

All meetings of the Council except the joint banquet on Saturday evening will be held in the Carnegie Institute of Technology.

Friday Evening, December 28, at Eight o'Clock
The Union Room, Administration Building

FIRST GENERAL SESSION

1. Address of Welcome.
2. Response, Professor W. D. Reeve.
3. Address—Dr. Vera Sanford, State Normal School, Oneonta, N. Y.
4. Symposium on methods of Making Mathematics Interesting.
   Teachers of Pittsburgh, directed by Dr. Elizabeth B. Cowley, President of the Mathematics Section, Pennsylvania State Education Association.

Saturday Morning, December 29, at Nine-thirty o'Clock

Room 345, Administration Building

General Topic

Mathematical Concepts of Value to High School Teachers

1. Concepts in Geometry
   Prof. H. W. Brinkman, Swarthmore College, Swarthmore, Pa.
2. Different Kinds of Equality
   Prof. C. C. MacDuffee, Ohio State University, Columbus, Ohio.

Special Feature

At the conclusion of this program Professor W. D. Cairns will give a short address introducing and explaining a display of English textbooks and examination papers which he collected while visiting English schools on his recent trip abroad.

Luncheon Meeting of Board of Directors at Twelve-thirty o'Clock

Saturday Afternoon, December 29, at Three-thirty o'Clock

Room 345, Administration Building

Joint Session with the Mathematical Association of America

Topic:

The Need for Re-Orientation of Mathematics in the Secondary Schools

1. From the Viewpoint of Modern Educational Theory
   Prof. P. W. Hutson, University of Pittsburgh.
2. From the Viewpoint of the University Professor of Mathematics
   Prof. W. L. Hart, University of Minnesota, Minneapolis, Minn.
3. From the Viewpoint of the High School Teacher
   Dr. M. L. Hartung, University High School, Madison, Wis.

Saturday Evening, December 29, at Six o'Clock

Hotel Webster Hall

Joint Banquet for all mathematicians, members of the National Council, Mathematical Association of America, and American Mathematical Society participating.

Book Review Department

Edited by
P. K. Smith

Analytical Geometry, by Vincent C. Poor. John Wiley & Sons, New York, 1934. iv+244 pages.

Although it is well known that vector analysis is extremely useful in simplifying the derivations of many of the equations of analytical geometry, few texts offer more than an explanation of the fundamentals. In the few in which use is made of these fundamentals in the development of the subject, it is the belief of the author that too little has been made of the opportunity. This, he considers, ample justification for the text.

Needless to say, even in this text the use of vectors is decidedly restricted but those topics in which it is used are well presented and should be easy for the student to understand. Among such topics are the division of a segment into two segments in given ratio, the normal form of the equation of a line in a plane, the normal form of the equation of a plane, the formula for the angle between two space lines, and equations of transformation.

There are many other good features in this book. Of these mention may be made of the careful presentations of oriented angles and direction cosines; the derivation of the equation of the tangent at a point on a conic, in which the $\frac{\Delta y}{\Delta x}$ method of calculus is used; and the shortening of the discussion on the central conics where the substitution of λ for $a^2(1-\theta^2)$ is made.

Higher plane curves and curve fitting are relegated to one chapter and may be omitted without loss of continuity, as is also the case with diameters, poles, and polars.

It is, of course, not entirely free from faults. For example we may criticize the statement on p. 26, line 2, that "As soon as the positives sense on one of these lines is chosen, an angle, θ, between the polar axis and the line is uniquely determined." Also, on p. 42, first sentence, "The term normal form has no further significance than standard form." On p. 145, (5.2), if $A=C$ we have division by zero so that this is better treated as a separate case. Also, improvement to clarify the subject matter might well be made in the second paragraph, p. 40, starting "If C is different from zero. . . . "

The printing and figures are good although one imperfection of a minor character was noticed, an exponent in equation 2°, p. 127.

The style of the book, its contents and general set-up are such as to make it a very good text. The discussions are concise and clear. The vector analysis will serve to freshen the course for the teacher who usually welcomes change in mode of presentation.

R. H. KNOX, JR.

First Course in The New Mathematics. Second Course in The New Mathematics. Elementary Algebra. By Edward I. Edgerton and Perry A. Carpenter. Allyn and Bacon, Boston, New York, Chicago, Atlanta, San Francisco, Dallas, 1934.

In these texts the authors have kept in mind "interest, simplicity, attractiveness, logical sequence, careful gradation, practical application and thoroughness."

Certainly their exteriors are very attractive. In a display of books, one could scarcely resist reaching for one of them as a very first choice. And why not? Aren't mathematics texts as much entitled to *attractive* exteriors as texts on other subjects?

Scanning the pages of these books, the reviewer is struck with their social value. The treatment of the material is thoroughly abreast of the times; chapter headings, section titles, and picture captions are strong and appropriate. An unusually large number of pictures fit into the material: 106 in the first volume, 100 in the second, and 57 in the third—pictures that suggest the "practical social application" of the accompanying exercises. Each volume contains an appendix or supplementary list of exercises and a good index.

The first volume is designed for use in the seventh grade, It contains v+448 pages. Integers, common fractions, fractions as ratios pictured in graphs, decimals, percentage, first steps in business interest (foundations of banking), and exercise of everyday business—these and other topics are simply and thoroughly presented. Intuitive geometry and foundation work for algebra are well included.

The second volume contains ix+394 pages and is designed for use in the junior high school and upper grammar grades. It is treated in two parts: "Time Saving" in Part I, 180 pages, is accomplished by "short-cuts," "equations and formulas," "ratio and proportion," "similar triangles," "powers and roots" and "time measurements." Part II, pages 181-348 treats "interest," "stocks and bonds," "insurance," "taxes," "business gains and losses (negative numbers)" and then gives a "gen-

eral survey of business." "Graphs, games, tests, and experimental problems" serve to arouse interest and create initiative.

Elementary Algebra covers vii+498+4 pages, and is broken into three parts: Part I, "Through Simple Equations," 182 pages; Part II, "Through Quadratic Equations," 227 pages; and Part III, "Through Numerical Trigonometry," 45 pages. Nearly 50 other pages of supplementary exercises and examination questions are added. The book presents algebra as "the science of the equation." Care is taken to have the subject develop logically out of the first two volumes and, further, to have each chapter follow logically the preceding one.

Graphs are given a "unique" treatment—they are of two kinds: Arithmetic graphs illustrating *statistical* matters, and *functional* graphs; geometry is given prominence; examples and problems are numerous; tests are featured; cumulative reviews are afforded; and apropriate pictures add a touch of life and social value.

This series will prove popular both with teachers of mathematics and their pupils. It merits a close examination and a careful trial.

—IRBY C. NICHOLS

Mother of the Sciences

"Religion is the mother of the sciences. When the children grew up they left their mother; philosophy stayed at home to comfort the lady in her old age. The long association told more on the daughter than on the mother.

To this day the central problems of philosophy smack of theology. It seems to me that what philosophy lacks most is a principle of relativity.

A principle of relativity is just a code of limitations: it defines the boundaries wherein a discipline shall move and frankly admits that there is no way of ascertaining whether a certain body of facts is the manifestation of the observata, or the hallucination of the observer." —"Number The Language of Science," by Tobias Dantzig.

Problem Department

Edited by

T. A. BICKERSTAFF

This department aims to provide problems of varying degrees of difficulty which will interest anyone who is engaged in the study of mathematics.

All readers, whether subscribers or not, are invited to propose problems and to solve problems here proposed.

Problems and solutions will be credited to their authors.

While it is our aim to publish problems of most interest to the readers, it is believed that regular text-book problems are, as a rule, less interesting than others. Therefore, other problems will be given preference when the space for problems is limited.

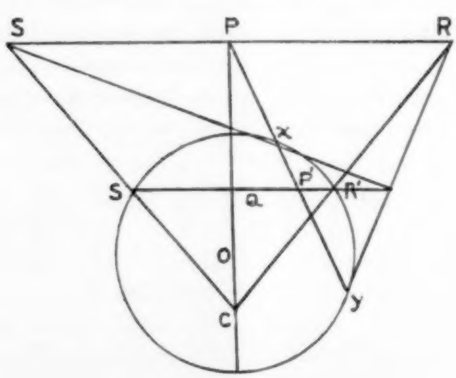
Send all communications about problems to T. A. Bickerstaff, University, Mississippi.

SOLUTIONS

No. 52. Proposed by H. T. R. Aude, Colgate University.

Given a circle with the center at O and a point P not on the circle. A line is drawn through P cutting the circle in two points at which the tangents are drawn. These meet the line through P which is perpendicular to PO in the points C and D. Are PC and PD equal in length?

A solution by projection by C. D. Smith, Mississippi State College.



Select P exterior to circle O and draw Pxy. Tangents at x and y determine segment SR $\perp$ to PO at P. Let P′ be harmonic conjugate of P relative to x and y. The locus of P′ is the polar of point P. This polar is $\perp$ to PO at Q and has contacts S′ and R′ with the circle. Likewise SR is the polar of Q and the two polars are conjugate lines. We now project R′ from R to C on PO so that from C as center P′ projects into a corresponding point of SR. Now corresponding tangents relative to Q meet on SR so that S′ corresponds to S and hence SS′ contains C. But Q is midpoint of S′R′ and hence the corresponding point P is the midpoint of SR since the midpoints are projective when the segments are parallel. For P interior to the circle a similar proof holds.

Another solution of number 43 has been found interesting. It is therefore given place here.

No. 43. Proposed by C. D. Smith, Mississippi State College.

Given a Quadrilateral P_1 P_2 P_3 P_4 with the usual rectangular coordinates. Find the conditions under which

$$x_1x_3-x_2x_4+y_1y_3-y_2y_4=0 \tag{1}$$

As solved by C. H. Sisam, Colorado College, Colorado Springs, Colorado.

If we write the above equation in the form

$$x_1x_3+y_1y_3=x_2x_4+y_2y_4$$

we may denote the value of these equal quantities by a^2 (wherein a^2 is positive, negative, or zero).

The two resulting equations

$$x_1x_3+y_1y_3=a^2 \text{ and } x_2x_4+y_2y_4=a^2$$

state that P_1, P_3 and P_2, P_4 are conjugate points (i. e., each lies on the polar line of the other) with respect to the (real, imaginary, or point) circle

$$x^2+y^2=a^2$$

with center at the origin. The existence of such a circle is thus the condition that (1) is satisfied.

If the radius a of the required circle is real, we may determine it as follows: Let O be the origin, P'_1 the foot of the perpendicular from P_1 on O P_3 and P'_2 the foot of the perpendicular from P_2 on

$O\,P_4$.* On $P'_1\,P_3$ and P'_2P_4 as diameters, draw circles and let T_1 be the point of tangency of a tangent from O to the first circle and T_2 of a tangent from O to the second. Then, the condition that (1) is satisfied is

$$O\,T_1 = O\,T_2 = a$$

For, if (r_i, Θ_i) are the polar coordinates of P_i, then

$$x_1x_3 + y_1y_3 = r_1r_3\,(\cos\Theta_1\cos\Theta_3 + \sin\Theta_1\sin\Theta_3) =$$

$$r_1r_3\cos(\Theta_1 - \Theta_3) = O\,P'_1\,.\,O\,P'_3 = O\,T^2_1 = a^2$$

and similarly for OT_2.

If a is imaginary, let us replace P_2 and P_4 by their symmetric points $\overline{P}_2$ and $\overline{P}_4$ with respect to O. Then the condition that (1) is satisfied for the given quadrilateral is that it also holds for the quadrilateral $P_1P_2\overline{P}_3\overline{P}_4$ and, for this latter quadrilateral, a is real.

Finally, if $a = 0$, the required condition is that $O\,P_1$ is perpendicular to $O\,P_3$ and $O\,P_2$ to OP_4.

It is obvious from the foregoing discussion that the truth of equation (1) depends, in general on the position of the origin. We shall show, in fact, that if $P_1P_2P_3P_4$ is any given quadrilateral (not a parallelogram with vertices taken in order) there exists a line such that, if the origin is taken on this line, equation (1) will hold. We shall further show that, if $P_1P_2P_3P_4$ are the vertices of a rectangle taken in order, then (1) holds for all positions of the origin.

Let, then, $P_1P_2P_3P_4$ be the vertices of any given quadrilateral. It is required to translate the origin from O to a point O' (h,k) such that, in the new coordinates (x', y'),

$$x'_1x'_3 - x'_2x'_4 + y'_1y'_3 - y'_2y'_4 = 0 \qquad (2)$$

We have, under such a translation,

$$x' = x - h \qquad\qquad y' = y - k$$

If we substitute these values of x' and y' in (2) and expand, we have, in the original coordinates x and y,

$$x_1x_3 - x_2x_4 + y_1y_3 - y_2y_4 - (x_1 + x_3 - x_2 - x_4)h - (y_1 + y_3 - y_2 - y_4)h = 0 \quad (3)$$

If, in this equation, we consider h and k as current coordinates of a point, and if the quantities

$$x_1 + x_3 - x_2 - x_4 \text{ and } y_1 + y_3 - y_2 - y_4$$

are not both zero, then (3) defines a line such that, if O′ lies on this line, equation (2) is satisfied.

If, however, we have simultaneously

$$x_1+x_3-x_2-x_4=0 \text{ and } y_1+y_3-y_2-y_4=0 \tag{4}$$

but

$$x_1+x_3-x_2\,x_4+y_1\,y_3-y_2\,y_4\neq 0 \tag{4'}$$

then no point O′ can be found such that (2) is satisfied. But the simultaneous equations (4) are the conditions that the points $P_1P_2P_3P_4$ are the vertices of a parallelogram taken in order.

If both equations (4) are satisfied and if the left member of (4′) is also equal to zero, then (2) is satisfied for all positions of the origin O′. But these three equations are precisely the conditions that the points $P_1P_2P_3P_4$ are the vertices of a rectangle taken in order.

PROBLEMS FOR SOLUTION

No. 69. Proposed by Alexander W. Boldyreff, University of Arizona.

Prove the following theorem:

Let R(x,y) be a rational function of x and y, where

$$y^n=a(x-b)^{n-1}(x-c),$$

a, b, c, being real constants and n a positive integer.

Then, $\int R(x,y)\,dx$ is integrable in terms of elementary functions.

No. 70. Proposed by Richard A. Miller, University of Mississippi.

Find the ellipse which will have a maximum area for a given perimeter.

No. 71. Proposed by Alexander W. Boldyreff:

Prove that $$\int_0^{\frac{\pi}{4}} \sec^{2n}x\,dx=\sum_{k=0}^{k=n-1} \frac{{}_{n-1}C_k}{2k+1}$$

No. 72. Proposed by Norman Anning, University of Michigan:

In a plane three straight lines are drawn through a point O in such a way that the whole angle around O is divided into six equal parts. A straight line not through O meets these three lines in A, B,

and C. Show that of the lengths OA, OB, OC, the reciprocal of one is equal to the sum of the reciprocals of the other two.

No. 73. Proposed by Alexander W. Boldyreff:

Prove that:

$$\frac{\sqrt{20+\sqrt{20+\sqrt{20+\ldots \text{ to } \infty}}}}{\sqrt{6+\sqrt{6+\sqrt{6+\ldots\ldots \text{ to } \infty}}}} = 3$$

No. 74. Proposed by H. T. R. Aude, Colgate University:

Prove that if the five numbers, a, b, c, d, e, are so related that $b=a+d$, $c=a+e$, and a is the geometric mean of d and e, then the sum $a^2+b^2+c^2$ is a square.

The following easily obtained but nevertheless very interesting results have been pointed out:

1. By Dewey C. Duncan, University of California, Berkeley, California.

The amount of one cent at 6 percent compounded annually for 1934 years is equivalent to 64,129,500,000,000,000 spheres of gold the size of the earth, assuming the radius of the earth to be 8000 miles and the value of a gold brick 8 by 2 by 1 inches to be 40,000 dollars.

2. By Harry Gwinner, University of Maryland:

Making no allowance for air resistance, a baseball must leave the hand of a pitcher in the pitcher's box at a distance of five and one-half feet from the ground at a rate of 138.56 feet per second in order to cross the home plate 30 inches above the ground, assuming that the ball is thrown horizontally.

By test, Walter Johnson threw a ball at the rate of 122 feet per second.

Mongo of the Brooklyn Dodgers threw a ball the distance from the mound to the plate in .533 seconds; and sports writers credit Lefty Grove with the ability to throw a ball 16.5 feet in one-tenth of a second.